Optimal Mechanism Design with Aftermarket Interactions

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Abstract

A revenue maximizing mechanism designer chooses a mechanism in the primary market to sell an object to privately informed entrants. The winning entrant then engages in Cournot competition with an incumbent in the aftermarket. The designer has perfect control in the primary market but imperfect control in the aftermarket. We fully characterize optimal mechanisms under general conditions. When the designer has "partial control" in the aftermarket, the constructed optimal mechanism is deterministic and the designer fully reveals the winning entrant's private production cost to the incumbent. When the designer has "no control" in the aftermarket, similar results hold.

Keywords: Adverse Selection, Aftermarket, Cournot Competition, Mechanism Design,

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Moral Hazard.

JEL Classifications: C72, D44, D82, D83, L12

1 introduction

Mechanism designers, who in real life could be firms, franchise companies, governments, patent owners, etc., often face players who will involve in certain aftermarket interactions. For example, after VoiceStream Wireless sold itself to Deutsche Telekom AG in 2001 for $35 billion and became T-Mobile USA, Inc. in July, 2002, it had to compete with other nationwide telecoms such as AT&T Mobility. When a new franchise for McDonald's is issued to an entrant, it needs to face competitors such as KFC in the fast food industry. When a government issues a licence to a firm to operate, this firm needs to interact with other firms already in the industry. When an owner of a cost reducing technology sells her patent to competing firms, those firms need to interact after the patent is acquired.

The common observation is that although the mechanism designers have relatively strong power in the primary market in determining how to sell the object, they usually have imperfect control over the aftermarket interactions. For instance, there is no evidence that Voicestream Wireless (or McDonald's) could intervene AT&T's (or KFC's) business decisions in the aftermarket. Nevertheless, the mechanism designers can influence the outcome in the aftermarket by revealing certain information obtained in the primary market to the aftermarket. For example, VoiceStream Wireless can choose whether to reveal the purchase price of $35 billion to the public. Similarly, when a franchise is auctioned off among the entrants, the bids contain the entrants' private information. As a result, the designer can at least choose whether or not to announce those bids to the public. Different announcement policies will result in different information updating, which may change players' incentives when they interact in the aftermarket, directly and indirectly affecting the mechanism designer's payoff.

Indeed, such considerations on aftermarket interactions have attracted ample attentions in the auction literature. Das Varma [5] examines first-price auctions for a cost reducing innovation bid by some oligopolists who will take part in aftermarket competition. The firms are privately informed about the amount of their production costs that the innovation can reduce. When the aftermarket competition is in Cournot style, there is a unique fully separating equilibrium; if it is in Bertrand style, a fully separating equilibrium may fail to exist. Goeree [6] examines first-price, second-price, and English auctions with abstract aftermarket competitions and compares their revenues. Scarpatetti and Wasser [16] allow multiple objects to be sold in the auctions. In Katzman and Rhodes-Kropf [9], what is allocated through the auction is the access to a duopoly with an incumbent firm.[1] The above strand of literature examines certain specific games and characterizes their equilibria. A natural next step, which is what we are taking in this paper, is to design an optimal mechanism in the presence of possible aftermarket interactions.

In this paper, we adopt a mechanism design approach. This allows us to examine one important issue that is not addressed in the above existing auction literature: how much information (regarding the bidders' bids) the auctioneer should reveal after the auction? The usual assumption in the existing literature is that the auctioneer announces only the transaction price (i.e., the highest bid in a first-price auction, or the second highest bid in a second-price auction). The question is whether this is optimal for the auctioneer. In

[1] There is also a strand of literature which focuses on the case of complete information. This includes Kamien et. al [7] and Katz and Shapiro [8].

theory, the auctioneer has many options. She can conceal all the information, reveal all the information, reveal information stochastically or partially, etc. Obviously, it is almost impossible to formulate all possible announcement rules one by one. Our paper considers all possible rules for information revelation and characterizes the optimal one. We find that in the optimal mechanism we will construct, full information revelation is the rule. This is a positive property since sometimes it may be hard for auctioneers to conceal information or to prohibit communications between the bidders.[2] Our result shows that there is no need to hide the information, and thus revealing all the information is revenue maximizing.

There could be many ways to model the aftermarket interactions. In this paper, we consider a stylized environment where a mechanism designer (franchise company, government, inventor, etc.) decides on how to sell an object (franchise, licence, patent, etc.) to a few potential entrants with privately known production costs. The market is currently occupied by an incumbent with a commonly known production cost. When a potential entrant wins the object, the aftermarket interaction is modeled as a Cournot competition between the winning entrant and the incumbent. When no potential entrant wins, the incumbent remains a monopoly. We assume that the designer has full control in the primary market and can determine the allocation rules, monetary transfers from the entrants, and how much information to reveal to the aftermarket. In contrast, we assume that the designer has only imperfect control in the aftermarket. For instance, the designer has no control over the incumbent at all; she can neither collect money from the incumbent nor dictate its production level in the aftermarket. Regarding the controlling power of the designer over the winning entrant in the aftermarket, we consider two different scenarios: partial control and no control, depending on whether the designer can dictate a production level for the

[2] For example, VoiceStream Wireless may be required by law to announce the purchasing price.

winning entrant.[3] For the franchise and licence cases, the partial control scenario is more applicable. For the Voicestream Wireless and patent cases, it is more reasonable to assume that Voicestream Wireless and the inventor have no control over T-mobile and the winning firm, and therefore, the no control scenario is more applicable.

We are able to explicitly characterize the optimal mechanisms under general conditions in both scenarios. In the partial control scenario, the constructed optimal mechanism is deterministic. The designer is willing to allocate the object to an entrant only if its production cost is lower than a cutoff, and this cutoff is increasing in both the market size and the incumbent's production cost. In this optimal mechanism, the designer fully reveals the winning entrant's reported private cost to the incumbent, and the revelation can be transmitted through the winning entrant's monetary transfer. The incumbent will infer exactly the winning entrant's production cost in the aftermarket competition. Notably, although we model the aftermarket as a Cournot competition, i.e., the winning entrant and the incumbent choose production levels simultaneously, the outcome in the aftermarket is the same as a modified Stackelberg competition under complete information with the winning entrant being the leader. In the no control scenario, similar results hold, and the outcome in the aftermarket coincides with a standard Cournot competition under complete information. Entry happens less often and the designer achieves a strictly less revenue in the no control scenario than in the partial control scenario. In addition, in the no control scenario, when there is a single potential entrant, it is never optimal for the designer to make a take-it-or-leave-it offer to the entrant; meanwhile, when there are multiple symmetric potential entrants, the optimal mechanism can be implemented by a first-price auction with a reserve price and together with the announcement of the winning bid.

[3] The reason why we call the first scenario partial control is that the designer can only make decisions for the winning entrant but not for the incumbent in the aftermarket.

In our analysis, the designer faces a mechanism design problem with hidden information, hidden actions and multiple agents, since the entrants have private production costs and the production levels in the aftermarket are not fully dictated by the designer. Myerson [14] establishes the revelation principle and formulates direct mechanisms under this general setting, even though explicitly characterizing the optimal mechanism is nontrivial. McAfee and McMillan [12] consider the optimal design of team mechanisms when players have privately known abilities and unobservable individual efforts in the team production.[4] They find that the designer can achieve the same revenue as if the moral hazard problem were not present. In contrast, our paper illustrates that when the designer has imperfect control over the aftermarket activities, her rent extraction ability is reduced even when the agents are risk neutral.

Our paper is related to Molnar and Virag [13]. Their paper is more general than ours in the sense that they also consider the case of Bertrand competition in the aftermarket. Other than that, there are three main differences. First, the market structures are different. In their paper, all firms participate in the primary market, while only entrants participate in ours. Second, the approaches are different. They look for an optimal mechanism within mechanisms with certain properties, while we utilize the revelation principle developed in Myerson [14] and obtain an optimal mechanism among all feasible mechanisms. Third, their analysis focuses on uniform distributions, while we can accommodate general distributions.

Our paper is related to the vast literature on regulation pioneered by Baron and Myerson [2] who consider the optimal way to regulate a monopoly with private production cost. Their analysis has been extended in various directions. For example, Blackorby and Szalay

[4]In the literature of optimal incentive contracts, McAfee and McMillan [11], and Laffont and Tirole [10] characterize the optimal auction for a contract with privately informed agents who later choose unobservable efforts. The focus of this literature is different from ours, since after the winning agent obtains the contract, only he has to choose an action and there are no interactions among the players.

[3] extends the model to accommodate two dimensional private information (production cost and capacity). Auriol and Laffont [1] compare a regulated monopoly with a duopoly. The regulation literature usually focuses on the role of private information. There is no moral hazard problem as firms' decisions are fully controlled by the regulator. While their models are applicable in many environments, imperfect regulations may arise due to high monitor costs and lack of essential information. Introducing moral hazard to these models is technically challenging.

The rest of the paper is organized as follows. In Section 2, we describe the model. In Section 3, we characterizes the optimal mechanism when the designer can dictate the winning entrant's production decision (i.e., the partial control scenario). In Section 4, we characterize the optimal mechanism when the designer cannot dictate the winning entrant's production decision (i.e., no control scenario). In Section 5, we conclude.

2 The model

We consider the environment where a revenue maximizing designer decides how to sell an object to I potential entrants in the primary market. After the primary market concludes, the winning entrant, if any, interacts with an incumbent in the aftermarket. As discussed extensively in the introduction, the designer does not always have perfect control over the aftermarket. For instance, the designer can neither dictate the production level for the incumbent nor collect monetary transfers from the incumbent. Regarding the entrant's behavior in the aftermarket, we focus on two different scenarios. In the first scenario, the designer can dictate the winning entrant's decisions in the aftermarket. In the second scenario, the designer cannot do so. We call the first scenario *partial control* since the designer has control of the entrant but not the incumbent, and the second one *no control* since the designer cannot control the aftermarket at all.[5] Although the designer cannot fully control the aftermarket, she can nevertheless influence its outcome by revealing certain information obtained in the primary market to the aftermarket. This changes the beliefs of the winning entrant and the incumbent about each other, and therefore, affects their decisions in the aftermarket. The question of how to optimally reveal the information to the aftermarket is an important question we aim to address. This is also the focus of our technical analysis in this paper.

In the primary market, the designer decides the allocation rules for the object, the entrants' transfer payments, and the information to be revealed to the aftermarket. We assume that the incumbent can observe which, if any, entrant obtains the object after the primary market concludes. If no entrant wins the object, the incumbent behaves as a monopoly in the aftermarket. Otherwise, the outcome in the aftermarket is determined by a Cournot competition between the incumbent and the winning entrant, where they simultaneously choose their production levels.

We assume that the winning entrant and the incumbent produce homogeneous product. The inverse demand function for this product is characterized by linear function $p = a - q$, where p denotes the market price, q denotes the total supply, and parameter a is a measure of the market size. All firms have constant marginal production costs. For notational simplicity, we assume that there is a single potential entrant. We will illustrate how to generalize the model to any number of potential entrants in Section 5. Since the incumbent has been in the market for a longer time than the entrant, the public has better information about the incumbent. In particular, we assume that the incumbent's production cost is commonly

[5]We can also study the case of full control. But it is less interesting, as it is similar to the standard literature on regulation.

known as c_I. The entrant's production cost C_E follows a distribution with *c.d.f.* $F_E(c_E)$, *p.d.f.* $f_E(c_E)$, and normalized support $\mathcal{C}_E = [0,1]$. This C_E is the private information of the entrant. Let c_E denote possible realizations of C_E. We assume that the market size is relative large so that both the incumbent and the winning entrant will produce a positive amount in the aftermarket in the mechanisms we constructed. More specifically, we assume

Assumption 1 $a > 3\max\{1, c_I\}$.

We assume that the reverse hazard rate function, $\frac{f_E(c_E)}{F_E(c_E)}$, is strictly decreasing. In this paper, without loss of generality, we normalize the designer's reservation value of the object to zero. We begin with the partial control scenario.

3 Partial control scenario

In the partial control scenario, the designer decides whether to allocate the object to the entrant, the payment from the entrant, as well as the level of production for the entrant and the amount of information to be revealed to the aftermarket. She can neither ask for payments from the incumbent nor dictate the incumbent's production level in the aftermarket. The designer's problem is a mechanism design problem with hidden information (private cost for the entrant), hidden actions (the incumbent's private production level in the aftermarket) and multiple agents. Formally, the designer offers a mechanism $\mathcal{M} \to R \times \Delta(\{0,1\} \times R_+ \times \Sigma)$ such that, when the entrant reports a message $m \in \mathcal{M}$, he pays $t_E(m) \in R$ and with density $\psi(x, q_E, \sigma)$ the following happen; the entrant either enters ($x = 1$) or stays out ($x = 0$), and the designer dictates a production level $q_E \in R_+$ for the entrant and reveals certain information $\sigma \in \Sigma$ to the aftermarket.[6]

We make use of the revelation principle developed in Myerson [14] and [15] (Section 6.3) throughout our analysis and restrict our search of the optimal mechanism to direct mechanisms without loss of generality. This revelation principle originally deals with discrete types, but can be extended to continuous types by changing summations to integrals in the derivations.[7] We can thus replicate the outcome induced by any indirect mechanism through a direct mechanism, where the message space is the type space and the information is transmitted through recommendations on actions that are not controlled by the designer. Specifically, a direct mechanism $\mathcal{C}_E \to R \times \Delta(\{0,1\} \times R_+ \times R_+)$ is such that when the entrant reports his production cost $c_E \in \mathcal{C}_E$ to the designer, he pays $t_E(c_E) \in R$ and with density $\pi(x, q_E, q_I|c_E)$ the following happen; the entrant either enters ($x = 1$) or stays out ($x = 0$), and the designer dictates a production level $q_E \in R_+$ for the entrant and sends a private recommendation about the production level $q_I \in R_+$ to the incumbent. Note that when the entrant stays out, there is no need to dictate a production level for the entrant. However, for notational simplicity, we include q_E in the probability function but let it be zero all the time. The market price is derived from the inverse demand function upon the realizations of the total outputs.

The designer chooses a mechanism $(\pi(x, q_E, q_I|c_E), t_E(c_E))$ to maximize her revenue subject to a set of feasibility constraints. Note that the incumbent does not have private information but does have private action, while the entrant has private information but no private action. As a result, the incentive compatibility constraint for the incumbent (IC_I) requires that, given that the entrant truthfully reports his cost in the primary market and follows

[6] Since the entrant has quasi-linear preferences, monetary transfers matter only in terms of expectation. Indeed, $t_E(m)$ can always be treated as the expected transfer.

[7] This is similar to Calzolari and Pavan [4] and Zhang and Wang [17], where the revelation principle is utilized to analyze models with resale.

the designer's dictation in the aftermarket, the incumbent will be obedient and follow the recommendations in the aftermarket. The incentive compatibility constraint for the entrant (IC_E) requires that, given that the incumbent follows the recommendations and the entrant follows the designer's dictation in the aftermarket, the entrant will report his cost truthfully in the primary market.

The participation constraint for the entrant (PC_E) requires that participating in the mechanism is better than the outside option, which is normalized to zero. There is no need to consider the participation constraint for the incumbent, since the designer can neither dictate the production level nor collect any money from him. In fact, when the incumbent receives the recommendation from the designer, he can always choose to ignore this information. As a result, the incumbent's participation constraint is always satisfied. Finally, $\pi(x, q_E, q_I|c_E)$ must be a valid probability distribution: $\forall q_E \in R_+, q_I \in R_+, x \in \{0,1\}, c_E \in \mathcal{C}_E$,

$$\pi(x, q_E, q_I|c_E) \geq 0 \; and \; \int_{R_+}\int_{R_+}\sum_{x=0}^{1}\pi(x, q_E, q_I|c_E) = 1. \tag{1}$$

The designer needs to maximize her revenue, i.e., the expected monetary transfers from the entrant, subject to feasibility constraints IC_I, IC_E, PC_E and (1). In the following subsections, we will examine these constraints one by one, starting backward from the aftermarket. The equilibrium concept we employ is perfect Bayesian Nash equilibria.

3.1 The aftermarket: establishing IC_I

Consider on-the-equilibrium path continuation games where the entrant has reported his production cost c_E truthfully, and the designer carries out his commitment to implement the mechanism $(\pi(x, q_E, q_I|c_E), t_E(c_E))$. When the entrant stays out, i.e., $x = 0$, the only incentive compatible recommendation for the incumbent is the monopoly level $q_I = \frac{a-c_I}{2}$. Note that we set $q_E = 0$ in this case although there is no need for the designer to dictate the entrant's production level.

When the entrant enters, i.e., $x = 1$, we can examine the Cournot competition in the aftermarket. Since the entrant's production level is dictated by the designer, we only need to examine the incumbent's incentive compatibility constraint in the aftermarket, i.e., IC_I. When the incumbent receives recommendation q_I, the incumbent needs to choose a production level $\tilde{q}_I$ to maximize his expected profit, i.e.,

$$\max_{\tilde{q}_I \geq 0} \int_{\mathcal{C}_E} \int_{R_+} \left\{ [a - \tilde{q}_I - q_E - c_I] \, \tilde{q}_I \right\} \pi(x = 1, q_E, q_I | c_E) f_E(c_E) dq_E dc_E \tag{2}$$

There are two types of uncertainty in the incumbent's payoff. First, the incumbent does not know c_E; second, conditional on c_E, the incumbent does not know the realization of the entrant's dictated production level q_E. As a result, the incumbent needs to form a belief. The information the incumbent has is that the entrant enters $x = 1$ and he receives recommendation q_I. The objective function (2) is strictly concave, and therefore, there exists a unique maximum. In equilibrium, the incumbent should obey the designer's recommendation, i.e., $\tilde{q}_I = q_I$, and the FOC yields the necessary and sufficient condition of the obedient condition for the incumbent. Thus, we obtain the following lemma,[8]

Lemma 1 *IC_I is satisfied if and only if, $\forall q_I$,*

$$q_I = \frac{a - c_I - \int_{\mathcal{C}_E} \int_{R_+} q_E \pi(x = 1, q_E, q_I | c_E) dq_E dF_E(c_E)}{2}$$

[8] Here we assume that the FOC yields an interior solution. As to be shown later, Assumption 1 does guarantee a positive production level for the incumbent in the optimal mechanism.

3.2 The primary market: establishing IC_E and PC_E

Now we examine the primary market. Note that only the entrant has private information and he is the only one who needs to report , i.e, IC_E. When the entrant reports $\tilde{c}_E$, the designer will implement mechanism $(\pi(x, q_E, q_I|\tilde{c}_E), t_E(\tilde{c}_E))$. The entrant anticipates that the designer will keep her commitment and that the incumbent will be obedient. Note that the entrant's production level is dictated by the designer in the aftermarket. Also note that the entrant earns a positive payoff only when he enters, i.e., $x = 1$. Therefore, knowing his true cost c_E, the entrant's payoff by reporting $\tilde{c}_E$ is given by

$$\begin{aligned} & U_E(c_E, \tilde{c}_E) \\ = \quad & \int_{R_+}\int_{R_+} \{[a - q_I - q_E - c_E]\, q_E \pi(x = 1, q_E, q_I|\tilde{c}_E)\}\, dq_E dq_I - t_E(\tilde{c}_E). \end{aligned} \tag{3}$$

The expectation is taken because the entrant does not know q_E and q_I when making decisions. The incentive compatibility constraint IC_E and participation constraints PC_E imply that

$$U_E(c_E, \tilde{c}_E) \leq U_E(c_E, c_E), \forall c_E, \tilde{c}_E \tag{4}$$

$$U_E(c_E, c_E) \geq 0, \forall c_E \tag{5}$$

As is common in the mechanism design literature, the envelope theorem yields

$$\begin{aligned} \frac{dU_E(c_E, c_E)}{dc_E} &= -\int_{R_+}\int_{R_+} q_E \pi(x = 1, q_E, q_I|c_E) dq_E dq_I \\ \Rightarrow \quad U_E(c_E, c_E) &= \int_{c_E}^{1}\int_{R_+}\int_{R_+} q_E \pi(x = 1, q_E, q_I|\xi) dq_E dq_I d\xi + U_E(1, 1) \end{aligned} \tag{6}$$

The following lemma shows that IC_E and PC_E are equivalent to the following conditions. The proof is standard and thus omitted.

Lemma 2 *IC_E and PC_E are satisfied if and only if the following conditions hold.* $\forall c_E \in \mathcal{C}_E$,

$$\begin{aligned} t_E(c_E) &= \int_{R_+}\int_{R_+}[a-q_I-q_E-c_E]q_E\pi(x=1,q_E,q_I|c_E)dq_Edq_I \\ &\quad -\int_{c_E}^{1}\int_{R_+}\int_{R_+} q_E\pi(x=1,q_E,q_I|\xi)dq_Edq_Id\xi - U_E(1,1), \end{aligned} \tag{7}$$

$$\int_{R_+}\int_{R_+} q_E\pi(x=1,q_E,q_I|c_E)dq_Edq_I \text{ is decreasing in } c_E \tag{8}$$

$$U_E(1,1) \geq 0 \tag{9}$$

The first condition simply rewrites the envelop condition (6), the second one is the monotonicity condition, and the third one is directly from PC_E with $c_E = 1$.

3.3 The designer's problem

Lemma 1 characterizes the equivalent conditions for IC_I; Lemma 2 characterizes the equivalent conditions for IC_E and PC_E. As a result, we can rewrite the designer's problem equivalently as

$$\max_{\pi(x,q_E,q_I|c_E),t_E(c_E)} \int_{\mathcal{C}_E} t_E(c_E)dF_E(c_E) \tag{10}$$

subject to:

$$q_I = \frac{a - c_I - \int_{\mathcal{C}_E} \int_{R_+} q_E \pi(x=1, q_E, q_I | c_E) dq_E dF_E(c_E)}{2}, \tag{11}$$

$$\begin{aligned} t_E(c_E) = & \int_{R_+} \int_{R_+} [a - q_I - q_E - c_E] q_E \pi(x=1, q_E, q_I | c_E) dq_E dq_I \\ & - \int_{c_E}^{1} \int_{R_+} \int_{R_+} q_E \pi(x=1, q_E, q_I | \xi) dq_E dq_I d\xi - U_E(1,1), \end{aligned} \tag{12}$$

$$\int_{R_+} \int_{R_+} q_E \pi(x=1, q_E, q_I | c_E) dq_E dq_I \textit{ is decreasing in } c_E \tag{13}$$

$$U_E(1,1) \geq 0 \tag{14}$$

$$\tag{1}$$

As is common in the literature, it is obvious that $U_E(1,1)$ should be set to zero. Substituting (11) and (12) into the objective function yields

$$\begin{aligned} & R_P \\ = & \int_{\mathcal{C}_E} t_E(c_E) dF_E(c_E) \\ = & \int_{\mathcal{C}_E} \left\{ \begin{array}{l} \int_{R_+} \int_{R_+} [a - q_I - q_E - c_E] q_E \pi(x=1, q_E, q_I | c_E) dq_E dq_I \\ - \int_{c_E}^{1} \int_{R_+} \int_{R_+} q_E \pi(x=1, q_E, q_I | \xi) dq_E dq_I d\xi, \end{array} \right\} dF_E(c_E) \quad (\textit{by Eqn.}\ (12)) \\ = & \int_{\mathcal{C}_E} \int_{R_+} \int_{R_+} [a - q_I - q_E - c_E - \frac{F_E(c_E)}{f_E(c_E)}] q_E \pi(x=1, q_E, q_I | c_E) dq_E dq_I dF_E(c_E) \end{aligned}$$

$$= \int_{\mathcal{C}_E}\int_{R_+}\int_{R_+}\left\{\begin{array}{l}\left[\begin{array}{l} a - q_E - c_E - \frac{F_E(c_E)}{f_E(c_E)} - \\ \frac{a - c_I - \int_{\mathcal{C}_E}\int_{R_+} q_E\pi(x=1,q_E,q_I|c_E)dq_E dF_E(c_E)}{2} \end{array}\right] \times \\ q_E\pi(x=1,q_E,q_I)\end{array}\right\} dq_E dq_I dF_E(c_E) \quad (by\ Eqn.\ (11))$$

$$= \int_{\mathcal{C}_E}\int_{R_+}\int_{R_+}\left[\frac{a}{2}+\frac{c_I}{2}-q_E-c_E-\frac{F_E(c_E)}{f_E(c_E)}\right] q_E\pi(x=1,q_E,q_I|c_E)dq_E dq_I dF_E(c_E)$$

$$+\frac{1}{2}\int_{R_+}\int_{\mathcal{C}_E}\int_{R_+}\left\{\begin{array}{l}\int_{\mathcal{C}_E}\int_{R_+} q_E\pi(x=1,q_E,q_I|c_E)dq_E dF_E(c_E)\times \\ q_E\pi(x=1,q_E,q_I|c_E)\end{array}\right\} dq_E dF_E(c_E) dq_I$$

$$= \int_{\mathcal{C}_E}\int_{R_+}\int_{R_+}\left[\frac{a}{2}+\frac{c_I}{2}-q_E-c_E-\frac{F_E(c_E)}{f_E(c_E)}\right] q_E\pi(x=1,q_E,q_I|c_E)dq_E dq_I dF_E(c_E)$$

$$+\frac{1}{2}\int_{R_+}\left\{\begin{array}{l}\int_{\mathcal{C}_E}\int_{R_+} q_E\pi(x=1,q_E,q_I|c_E)dq_E dF_E(c_E) \\ \int_{\mathcal{C}_E}\int_{R_+} q_E\pi(x=1,q_E,q_I|c_E)dq_E dF_E(c_E)\end{array}\right\} dq_I$$

$$\leq \int_{\mathcal{C}_E}\int_{R_+}\int_{R_+}\left\{\left[\frac{a}{2}+\frac{c_I}{2}-q_E-c_E-\frac{F_E(c_E)}{f_E(c_E)}\right] q_E\pi(x=1,q_E,q_I|c_E)\right\} dqE dq_I dF_E(c_E)$$

$$+\frac{1}{2}\int_{R_+}\left\{ \int_{\mathcal{C}_E}\int_{R_+} q_E^2\pi(x=1,q_E,q_I|c_E)dq_E dF_E(c_E) \right\} dq_I$$

$$= \int_{\mathcal{C}_E}\int_{R_+}\int_{R_+}\left[\frac{a}{2}+\frac{c_I}{2}-\frac{q_E}{2}-c_E-\frac{F_E(c_E)}{f_E(c_E)}\right] q_E\pi(x=1,q_E,q_I|c_E)dq_E dq_I dF_E(c_E) \tag{15}$$

The right hand side of the inequality corresponds to the situation where the entrant's production cost is fully revealed to the incumbent. For the right hand side of (15), point-wise maximization can be applied. Conditional on c_E, let us first consider the term

$$\left[\frac{a}{2}+\frac{c_I}{2}-\frac{q_E}{2}-c_E-\frac{F_E(c_E)}{f_E(c_E)}\right] q_E$$

Since $q_E \geq 0$, this term is maximized by setting

$$q_E = \max\{\frac{a}{2} + \frac{c_I}{2} - c_E - \frac{F_E(c_E)}{f_E(c_E)}, 0\}$$

As a result,

$$\begin{aligned}
& R_P \\
\leq\ & \int_{\mathcal{C}_E} \int_{R_+} \int_{R_+} \max\left\{\frac{a}{2} + \frac{c_I}{2} - c_E - \frac{F_E(c_E)}{f_E(c_E)}, 0\right\}^2 \pi(x=1, q_E, q_I | c_E) dq_E dq_I dF_E(c_E) \\
=\ & \int_{\mathcal{C}_E} \max\left\{\frac{a}{2} + \frac{c_I}{2} - c_E - \frac{F_E(c_E)}{f_E(c_E)}, 0\right\}^2 \int_{R_+} \int_{R_+} \pi(x=1, q_E, q_I | c_E) dq_E dq_I dF_E(c_E) \\
\leq\ & \int_{\mathcal{C}_E} \left\{ \max\left\{\frac{a}{2} + \frac{c_I}{2} - c_E - \frac{F_E(c_E)}{f_E(c_E)}, 0\right\}^2 \int_{R_+} \int_{R_+} \sum_{x=0}^{1} \pi(x, q_E, q_I | c_E) dq_E dq_I \right\} dF_E(c_E) \\
=\ & \int_{\mathcal{C}_E} \max\left\{\frac{a}{2} + \frac{c_I}{2} - c_E - \frac{F_E(c_E)}{f_E(c_E)}, 0\right\}^2 dF_E(c_E) \quad (by\ Eqn.\ (1))
\end{aligned} \tag{16}$$

Note that the term $\frac{a}{2} + \frac{c_I}{2} - c_E - \frac{F_E(c_E)}{f_E(c_E)}$ is strictly decreasing in c_E, and is strictly positive at $c_E = 0$. We let $\hat{c}_E$ be the point that the above term crosses zero if $\frac{a}{2} + \frac{c_I}{2} - 1 - \frac{1}{f_E(1)} < 0$, and be 1 if $\frac{a}{2} + \frac{c_I}{2} - 1 - \frac{1}{f_E(1)} \geq 0$.

Define

$$J_E(c_E) = \left[\frac{a}{2} + \frac{c_I}{2} - c_E - \frac{F_E(c_E)}{f_E(c_E)}\right]^2 \tag{17}$$

It measures the marginal revenue of issuing the object to the entrant and fully revealing the entrant's production cost to the incumbent. Thus,

$$R_P \leq \int_0^{\hat{c}_E} J_E(c_E) dF_E(c_E) \tag{18}$$

As a result, we have established an upper bound revenue for the designer. If we can construct a feasible mechanism that achieves this upper bound revenue, then it will be an optimal mechanism. The following proposition shows that this upper bound revenue is always achievable.

Proposition 1 *In the partial control scenario, the following mechanism maximizes the designer's expected revenue.*

(i) Allocation rule and production decision for the entrant:

$$x = \begin{cases} 1, & if\ 0 \leq c_E \leq \hat{c}_E; \\ 0, & if\ \hat{c}_E < c_E \leq 1; \end{cases} \tag{19}$$

$$q_E = \frac{a}{2} + \frac{c_I}{2} - c_E - \frac{F_E(c_E)}{f_E(c_E)} \quad if\ 0 \leq c_E \leq \hat{c}_E \tag{20}$$

(ii) Aftermarket production recommendation for the incumbent:

$$q_I = \begin{cases} \frac{a-3c_I+2c_E+\frac{2F_E(c_E)}{f_E(c_E)}}{4} & if\ 0 \leq c_E \leq \hat{c}_E \\ \frac{a-c_I}{2} & if\ \hat{c}_E \leq c_E \leq 1 \end{cases} \tag{21}$$

(iii) The entrant's transfer payment to the designer:

$$t_E(c_E) = \begin{cases} \left[\frac{a}{2} + \frac{c_I}{2} - c_E - \frac{F_E(c_E)}{f_E(c_E)}\right]^2 - \int_{c_E}^{\hat{c}_E} \left[\frac{a}{2} + \frac{c_I}{2} - \xi - \frac{F_E(\xi)}{f_E(\xi)}\right] d\xi & if\ 0 \leq c_E \leq \hat{c}_E \\ 0 & if\ \hat{c}_E \leq c_E \leq 1 \end{cases}, \tag{22}$$

(iv) The designer's revenue:

$$R_P = \int_0^{\hat{c}_E} J_E(c_E) dF_E(c_E) \tag{23}$$

Proof: It is easy to verify that the above mechanism generates the upper bound revenue. We only need to prove that it satisfies the feasibility constraints. Since the aftermarket production recommendation for the incumbent is a strictly decreasing function of c_E, when the entrant enters, the incumbent will infer exactly the entrant's production cost. Thus, it is straightforward to verify that IC_I is satisfied. According to Lemma 2, for IC_E and PC_E, we only need to verify the monotonicity condition (7), which is trivially satisfied. Therefore, the proposed mechanism is feasible, and this completes the proof. **Q.E.D.**

There are many properties of this optimal mechanism, which are summarized in the following corollaries. For example, whether to allocate the object to the entrant is a cutoff rule, and the production levels in the aftermarket are in pure strategies. We have,

Corollary 1 *The constructed optimal mechanism is deterministic.*

The above corollary suggests that it is without loss of generality to focus on deterministic mechanisms in search of the optimal mechanism. The is important, because as to be shown in the no control scenario, it is easy and straightforward to lay out the model and the analysis with deterministic mechanisms, which suggests that solving the optimal deterministic mechanisms will be a very useful first step in tackling similar problems. We examine the cutoff $\hat{c}_E$ and obtain some comparative statics.

Corollary 2 *Entry happens more often when the market size is larger or when the incum-*

bent's production cost is higher.

Note that the designer can raise money only through the entrant. When a or c_I is larger, there are more profits to extract or the entrant is in a more advantageous position in the aftermarket. Therefore, the designer is willing to let the entrant to enter more often. If we examine Eqn. (21), it reveals that the recommendation to the incumbent is a one-to-one mapping function of the entrant's production cost when the entrant enters. This implies that after the incumbent learns the recommendation from the designer, he will infer exactly the entrant's production cost. We thus conclude

Corollary 3 *In the constructed optimal mechanism, the designer fully reveals the entrant's production cost to the incumbent.*

The intuition is subtle for this corollary. When the entrant himself, instead of the designer, can *ex-ante* commit how to reveal his information to the incumbent upon its realization, according to Blackwell's Theorem, it is beneficial to reveal all the information. The Blackwell's theorem does not apply when it is the designer to decide how much information to be revealed, since the designer's payoff is not convex in the entrant's distribution. However, the designer has the same interest as the entrant since her revenue is raised only from the entrant. Therefore, it is optimal for the designer to reveal the information on behalf of the entrant.

If we examine the aftermarket production levels upon entry, i.e., (20) and (21), it can be shown that this outcome is the same as a modified Stackelberg competition between the entrant and the incumbent under complete information, where the entrant is the leader and the incumbent is the follower. This is because the designer proposes mechanisms and makes production decision on behalf of the entrant, and therefore, can have some first mover advantage. However, the leader's cost is $c_E + F_E(c_E)/f(c_E)$ instead of c_E since the designer has to give informational rent to the entrant.

Note that the monetary transfer function is a strictly decreasing function of c_E and can fully reveal the entrant's private cost. As a result, upon seeing the transaction price for the object, the incumbent will infer exactly the incumbent's production cost. Thus, we have

Corollary 4 *To implement the optimal mechanism in practice, the designer only needs to announce the transaction price for the object to the aftermarket, and does not need to make production recommendations.*

4 No control scenario

In the no control scenario, the designer decides whether to allocate the object to the entrant and the payment, and how much information to reveal to the aftermarket; she can neither ask for payments from the incumbent nor dictate production levels for the incumbent nor the entrant in the aftermarket. Again, the revelation principle allows us to focus on direct mechanisms. As shown in the above section, the optimal mechanism is deterministic. This also holds in the no control scenario. Instead of going through the general setup, we will execute our analysis by focusing on the deterministic mechanisms, since it is much easier and more intuitive to lay out the model with deterministic mechanisms. With deterministic mechanisms, the only uncertainty in the model is the entrant's production cost and we can focus on its updating.

Formally, a direct deterministic mechanism $\mathcal{C}_E \to R \times \Delta(\{0,1\}) \times R_+^2$ is such that when the entrant reports his cost $c_E \in \mathcal{C}_E$ to the designer, he pays $t_E(c_E) \in R$, enters with probability $x_E(c_E)$, and the designer sends private recommendations about the production levels

$q_E(c_E) \in R_+$ and $q_I(c_E) \in R_+$ to the entrant and the incumbent in the Cournot competition upon the entrant's entry, respectively.[9,10] Since the entry probabilities and recommendations depend on c_E, they are also signals of the entrant's production cost.

The designer chooses a mechanism (x_E, q_E, q_I, t_E) to maximize her revenue subject to a set of feasibility constraints. Note that the incumbent does not have private information. The incentive compatibility constraint for the incumbent is only for the aftermarket (IC_I^A). It requires that, given that the entrant truthfully reports his cost in the primary market and follows the recommendation in the aftermarket, the incumbent will be obedient and follow the recommendation in the aftermarket. The incentive compatibility constraint for the entrant requires that, given that the incumbent follows the recommendation in the aftermarket, the entrant will report his cost truthfully in the primary market and be obedient in the aftermarket. We can break the entrant's incentive compatibility constraints into two parts. The first part (IC_E^A) is that, if the entrant has truthfully reported his cost in the primary market, it is optimal for him to follow the designer's recommendation in the aftermarket. The second part (IC_E^P) is that, the entrant will truthfully report his cost in the primary market given that he will behave optimally in the aftermarket. The participation constraint for the entrant (PC_E) is the same as in the partial control scenario. For the same reason, there is no participation constraint for the incumbent. Finally, since there is only one object to be allocated, the following additional condition must be satisfied:

$$0 \leq x(c_E) \leq 1, \forall c_E \tag{24}$$

The designer needs to maximize her revenue, i.e., the monetary transfers from the entrant,

[9] Here we actually allow more general mechanisms than deterministic mechanisms since the allocation rule can be stochastic. The key is that the recommendations are deterministic.

[10] When the entrant stays out, there is no need to make any recommendation for the entrant, and the only incentive compatible recommendation for the incumbent is the monopoly level of output.

subject to feasibility constraints $IC_I^A, IC_E^P, IC_E^A, PC_E$ and (24).

In the following subsections, we will examine these constraints one by one, starting backward from the aftermarket. The equilibrium concept we employ is perfect Bayesian Nash equilibrium. The no control scenario is technically more challenging than the partial control scenario. This is because the entrant has more ways to deviate by misreporting in the primary market and disobeying the recommendation from the designer in the aftermarket at the same time. In this case, it is usually hard to pin down a necessary and sufficient condition for the entrant's incentive compatibility constraint.

4.1 The aftermarket

4.1.1 The on-equilibrium-path continuation game: establishing IC_I^A and IC_E^A

Consider the Cournot competition in the aftermarket. Let us first examine the incumbent's incentive compatibility constraint in the aftermarket, i.e., IC_I^A. Along the on-the-equilibrium-path continuation game where the entrant reports his production cost c_E truthfully, the designer carries out his commitment and implements the mechanism $(x_E(c_E), q_E(c_E), q_I(c_E), t_E(c_E)$ With deterministic mechanisms, from the prospective of the incumbent, the uncertainty in his payoff in the Cournot competition only depends on the entrant's production cost. Let $\mathcal{Q}$ denote the image of $q_I(c_E)$, i.e., all possible realizations of equilibrium recommendations for the incumbent. For notational simplicity, we assume that the set $\mathcal{Q}$ is finite with N elements, and therefore, $\mathcal{Q} = \{q_I^1, \cdots, q_I^N\}$.[11] When the incumbent learns the entrant's entry and receives recommendation q_I^n, where $n \in \{1, \cdots, N\}$, let $g_I^n(c_E)$ denote the *p.d.f.* of the incumbent's updated belief about the entrant's production cost. Note that potentially many

[11]Later, it will be clear that this does not affect our results at all.

different values of c_E could lead to the recommendation q_I^n. We denote the set of production cost c_E that could lead to recommendation q_I^n as $\mathcal{C}_E^n = \{c_E : q_I(c_E) = q_I^n\}$.

We can now compute $g_I^n(c_E)$ explicitly. Since $x_E(c_E)$ and $q_I(c_E)$ are independent, the incumbent can update separately. Observing that the entrant enters the market, the incumbent updates his belief about the entrant's type to *p.d.f.*

$$h(c_E) = \frac{x_E(c_E) f_E(c_E)}{\int_{\mathcal{C}_E} x_E(c_E) f_E(c_E) dc_E} \tag{25}$$

Now the further information of recommendation q_I^n will lead to another updating. The incumbent knows that any production cost in $\mathcal{C}_E^n$ can leads to recommendation q_I^n. As a result, the incumbent updates his belief about the entrant's type further to

$$g_I^n(c_E) = \frac{h(c_E)}{\int_{\mathcal{C}_E^n} h(c_E) dc_E} \tag{26}$$

The incumbent maximizes his expected profit using the above updated *p.d.f*:

$$\max_{\tilde{q}_I \geq 0} \int_{\mathcal{C}_E^n} \left[a - \tilde{q}_I - q_E(c_E) - c_I\right] \tilde{q}_I g_I^n(c_E) dc_E \tag{27}$$

Since the objective function is strictly concave, there exists a unique maximum. In equilibrium, the incumbent should obey the designer's recommendation, i.e., $\tilde{q}_I = q_I^n$, and the FOC yields the obedient condition for the incumbent,

$$q_I^n = max\left\{\frac{a - c_I - \int_{\mathcal{C}_E^n} q_E(c_E) g_I^n(c_E) dc_E}{2}, 0\right\} \tag{28}$$

Now consider the entrant's incentive compatibility constraint in the aftermarket, i.e.,

IC_E^A. Suppose the entrant's cost c_E is in $\mathcal{C}_I^n$ and he truthfully reports it to the designer. The entrant anticipates that the incumbent will obey the designer's recommendation to produce q_I^n. There is no uncertain in the entrant's payoff. As a result, the entrant maximizes his expected payoff conditional on observing his own entry $x = 1$, his own production cost c_E, and recommendation $q_E(c_E)$:

$$\max_{\tilde{q}_E \geq 0} \left[a - q_I^n - \tilde{q}_E - c_E\right] \tilde{q}_E \tag{29}$$

Note that the objective function is strictly concave, and therefore, there exist a unique maximum. In equilibrium, the entrant should obey the designer's recommendation, i.e. $\tilde{q}_E = q_E(c_E)$, and the FOC yields the obedient condition for the entrant: $\forall c_E \in \mathcal{C}_E^n$,

$$q_E(c_E) = max\left\{\frac{a - c_E - q_I^n}{2}, 0\right\} \tag{30}$$

From (28) and (30), we can solve the incentive compatible recommendations for each firm in the following lemma.

Lemma 3 *IC_I^A and IC_E^A are satisfied, if and only if* $\forall c_E \in \mathcal{C}_E^n$, $\forall n \in \{1, \cdots, N\}$

$$q_I^n = \frac{a}{3} - \frac{2}{3}c_I + \frac{\mathbf{E}_{g_I^n}\left\{c_E\right\}}{3} \tag{31}$$

$$q_E(c_E) = \frac{a}{3} + \frac{1}{3}c_I - \frac{c_E}{2} - \frac{\mathbf{E}_{g_I^n}\left\{c_E\right\}}{6}, \tag{32}$$

where $\mathbf{E}_{g_I^n}\{(\cdot)\} = \int_{\mathcal{C}_E^n}(\cdot)g_I^n(c_E)dc_E$.

Proof: We first focus on interior solutions. Substituting (30) into (28) yields

$$
\begin{aligned}
q_I^n &= \frac{a - c_I - \mathbf{E}_{g_I^n}\left\{\frac{a-q_I^n-c_E}{2}\right\}}{2} \\
&= \frac{a - c_I - \frac{a-q_I^n-\mathbf{E}_{g_I^n}\left\{c_E\right\}}{2}}{2} \\
\Leftrightarrow \qquad & q_I^n = \frac{a}{3} - \frac{2}{3}c_I + \frac{\mathbf{E}_{g_I^n}\left\{c_E\right\}}{3} \qquad (33)
\end{aligned}
$$

Substituting the above equation into (30) yields the formula for $q_E(c_E)$. Note that with Assumption 1, the solution is indeed an interior solution. Assumption 1 also guarantees that this is the unique solution. **Q.E.D.**

4.1.2 A deviation

In order to determine the incentive compatibility constraints in the primary market, we need to know when the entrant reports his valuation to be $\tilde{c}_E \neq c_E$, how he would act in the after-market. In this case, he knows that the incumbent will obey the designer's recommendation and produce $q_I(\tilde{c}_E)$. Therefore, the entrant's problem in the Cournot competition is given by:

$$\max_{\tilde{q}_E \geq 0} \left[a - q_I(\tilde{c}_E) - \tilde{q}_E - c_E\right] \tilde{q}_E \qquad (34)$$

Note that the objective function is strictly concave, and therefore, there exists a unique maximum, and the FOC yields,[12]

$$\tilde{q}_E = \frac{a - q_I(\tilde{c}_E) - c_E}{2} \tag{35}$$

We thus have the following lemma.

Lemma 4 *When the entrant reports $\tilde{c}_E$ in the primary market, he will choose a production level $\frac{a-q_I(\tilde{c}_E)-c_E}{2}$ in the aftermarket.*

4.2 The primary market: Establishing IC_E^P and PC_E

Now we examine the primary market. Note that only the entrant has private information and he is the only one who needs to report (IC_E^P). Given that the incumbent is obedient and the entrant chooses the production level in the competition optimally according to Lemma 4, the entrant's payoff by reporting $\tilde{c}_E$ is

$$\begin{aligned}
& U_E(c_E, \tilde{c}_E) \\
= & \left\{\left[a - q_I(\tilde{c}_E) - \frac{a - q_I(\tilde{c}_E) - c_E}{2} - c_E\right] \frac{a - q_I(\tilde{c}_E) - c_E}{2}\right\} x_E(\tilde{c}_E) - t_E(\tilde{c}_E) \\
= & \left[\frac{a - q_I(\tilde{c}_E) - c_E}{2}\right]^2 x_E(\tilde{c}_E) - t_E(\tilde{c}_E). \qquad (36)
\end{aligned}$$

The incentive compatibility IC_E^P and participation constraints PC_E imply that

$$U_E(c_E, \tilde{c}_E) \leq U_E(c_E, c_E), \forall c_E, \tilde{c}_E \tag{37}$$

[12] Assumption 1 guarantees that the solution is interior.

$$U_E(c_E, c_E) \geq 0, \forall c_E \tag{38}$$

As common in the mechanism design literature, the envelope theorem yields

$$\begin{aligned}
\frac{dU_E(c_E, c_E)}{dc_E} &= 2\left[\frac{a - q_I(\tilde{c}_E) - c_E}{2}\right](-\frac{1}{2})x_E(\tilde{c}_E)\bigg|_{\tilde{c}_E = c_E} \\
&= -q_E(c_E)x_E(c_E) \\
&\Rightarrow \; U_E(c_E, c_E) = \int_{c_E}^{1} q_E(\xi)x_E(\xi)d\xi + U_E(1,1) \qquad (39)
\end{aligned}$$

As a result, IC_E^P and PC_E imply the following lemma.

Lemma 5 *IC_E^P and PC_E are satisfied only if the following conditions hold:*

$$t_E(c_E) = q_E(c_E)^2 x_E(c_E) - \int_{c_E}^{1} q_E(\xi)x_E(\xi)d\xi - U_E(1,1), \tag{40}$$

$$U_E(1,1) \geq 0 \tag{41}$$

The first condition is simply a rewrite of (39), and the second is directly from PC_E with $c_E = 1$.

4.3 The designer's problem

Lemma 3 characterizes the equivalent conditions for IC_I^A and IC_E^A; Lemma 5 characterizes the necessary conditions for IC_E^P and PC_E. In contrast to the standard literature, a necessary and sufficient condition for IC_E^P and PC_E cannot be obtained, and we cannot solve the problem directly. Our approach is to study a relaxed problem of the original problem and work out the optimal mechanism there. We then prove that this mechanism is also feasible in the original problem and is therefore optimal in the original problem. We can formulate the relaxed problem as follows:

$$\max_{q_I,q_E,x_E,t_E} \int_0^1 t_E(c_E)dF_E(c_E) \tag{42}$$

subject to:

$$q_E(c_E) = \frac{a}{3} + \frac{1}{3}c_I - \frac{c_E}{2} - \frac{1}{6}\mathbf{E}_{g_I^n}\{c_E\}, \tag{43}$$

$$t_E(c_E) = q_E(c_E)^2 x_E(c_E) - \int_{c_E}^1 q_E(\xi)x_E(\xi)d\xi - U_E(1,1), \tag{44}$$

$$U_E(1,1) \geq 0 \tag{45}$$

$$0 \leq x_E(c_E) \leq 1 \tag{46}$$

The reason why this problem is a relaxed problem of the original problem is as follows. First, the object functions are the same. Second, the feasibility constraints are implied by those in the original problem Lemma 3 and Lemma 5, and are therefore less restrictive. As a result, the solution provides an upper bound revenue for the original problem.

As is common in the literature, it is obvious that $U_E(1,1)$ should be set to zero. Substituting (43) and (44) into the objective function, the designer's problem becomes

$$\begin{aligned}
& R_N \\
= & \int_{\mathcal{C}_E} \left[q_E(c_E)^2 x_E(c_E) - \int_{c_E}^1 q_E(\xi)x_E(\xi)d\xi \right] dF_E(c_E) \quad (byEqn.\ (44)) \\
= & \int_{\mathcal{C}_E} \left[q_E(c_E)^2 x_E(c_E) - q_E(c_E)x_E(c_E)\frac{F_E(c_E)}{f_E(c_E)} \right] dF_E(c_E)
\end{aligned}$$

$$
\begin{aligned}
&= \int_{\mathcal{C}_E} \left[q_E(c_E) - \frac{F_E(c_E)}{f_E(c_E)} \right] q_E(c_E) x_E(c_E) f_E(c_E) dc_E \\
&= \sum_{n=1}^{N} \int_{\mathcal{C}_E^n} \left\{ \begin{array}{l} \left[\frac{a}{3} + \frac{1}{3}c_I - \frac{c_E}{2} - \frac{\mathbf{E}_{g_I^n}\left\{c_E\right\}}{6} - \frac{F_E(c_E)}{f_E(c_E)} \right] \times \\ \left[\frac{a}{3} + \frac{1}{3}c_I - \frac{c_E}{2} - \frac{\mathbf{E}_{g_I^n}\left\{c_E\right\}}{6} \right] x_E(c_E) f_E(c_E) \end{array} \right\} dc_E \quad (by\ Eqn.\ (43)) \\
&= \sum_{n=1}^{N} \int_{\mathcal{C}_E^n} \left\{ \begin{array}{l} \left[\frac{a}{3} + \frac{1}{3}c_I - \frac{c_E}{2} - \frac{\mathbf{E}_{g_I^n}\left\{c_E\right\}}{6} - \frac{F_E(c_E)}{f_E(c_E)} \right] \times \\ \left[\frac{a}{3} + \frac{1}{3}c_I - \frac{c_E}{2} - \frac{\mathbf{E}_{g_I^n}\left\{c_E\right\}}{6} \right] \times \\ \int_0^1 x_E(c_E) f_E(c_E) dc_E \int_{\mathcal{C}_E^n} h(c_E) dc_E g^n(c_E) \end{array} \right\} dc_E \quad (by\ Eqn.\ (26)) \\
&= \sum_{n=1}^{N} \int_0^1 x_E(c_E) f_E(c_E) dc_E \int_{\mathcal{C}_E^n} h(c_E) dc_E \times \\
&\quad \times \underbrace{\mathbf{E}_{g_I^n}\left\{ \left[\frac{a}{3} + \frac{1}{3}c_I - \frac{c_E}{2} - \frac{\mathbf{E}_{g_I^n}\left\{c_E\right\}}{6} - \frac{F_E(c_E)}{f_E(c_E)} \right] \left[\frac{a}{3} + \frac{1}{3}c_I - \frac{c_E}{2} - \frac{\mathbf{E}_{g_I^n}\left\{c_E\right\}}{6} \right] \right\}}_{denoted\ as\ W} \quad (47)
\end{aligned}
$$

Let us first examine the function W.

$$
\begin{aligned}
W &= \mathbf{E}_{g_I^n}\left\{ \left[\frac{a}{3} + \frac{c_I}{3} - \frac{c_E}{2} - \frac{F_E(c_E)}{f_E(c_E)} \right] \left[\frac{a}{3} + \frac{c_I}{3} - \frac{c_E}{2} \right] \right\} \\
&- \left(\frac{2}{3} + \frac{2c_I}{3} \right) \frac{\mathbf{E}_{g_I^n}\{c_E\}}{6} + \frac{\mathbf{E}_{g_I^n}\{c_E\} \mathbf{E}_{g_I^n}\left\{ c_E + \frac{F_E(c_E)}{f_E(c_E)} \right\}}{6} + \frac{\mathbf{E}_{g_I^n}\{c_E\} \mathbf{E}_{g_I^n}\{c_E\}}{36} \quad (48)
\end{aligned}
$$

We need the following lemma to proceed further.

Lemma 6 *(Majorization Inequality) Suppose $h_I'(c_E), h_E'(c_E) \geq 0$, then*

$$E[(h_I(c_E) h_E(c_E))] \geq E[h_I(c_E)] E[h_E(c_E)].$$

Therefore, according to the above Majorization Inequality, we obtain

$$
\begin{aligned}
W \quad &\leq \quad \mathbf{E}_{g_I^n}\left\{\left[\frac{a}{3}+\frac{c_I}{3}-\frac{c_E}{2}-\frac{F_E(c_E)}{f_E(c_E)}\right]\left[\frac{a}{3}+\frac{c_I}{3}-\frac{c_E}{2}\right]\right\} \\
&- \quad \left(\frac{2}{3}+\frac{2c_I}{3}\right)\frac{\mathbf{E}_{g_I^n}\{c_E\}}{6}+\frac{\mathbf{E}_{g_I^n}\left\{c_E[c_E+\frac{F_E(c_E)}{f_E(c_E)}]\right\}}{6}+\frac{\mathbf{E}_{g_I^n}\{{c_E}^2\}}{36} \\
&= \quad \mathbf{E}_{g_I^n}\left\{\left[\frac{a}{3}+\frac{1}{3}c_I-\frac{c_E}{2}-\frac{c_E}{6}-\frac{F_E(c_E)}{f_E(c_E)}\right]\left[\frac{a}{3}+\frac{1}{3}c_I-\frac{c_E}{2}-\frac{c_E}{6}\right]\right\} \\
&= \quad \mathbf{E}_{g_I^n}\left\{\left[\frac{a}{3}+\frac{1}{3}c_I-\frac{2c_E}{3}-\frac{F_E(c_E)}{f_E(c_E)}\right]\left[\frac{a}{3}+\frac{1}{3}c_I-\frac{2c_E}{3}\right]\right\} \qquad (49)
\end{aligned}
$$

Hence,

$$
\begin{aligned}
R_N \quad &\leq \quad \sum_{n=1}^{N}\int_0^1 x_E(c_E)f_E(c_E)dc_E\int_{\mathcal{C}_E^n}h(c_E)dc_E \times \\
&\quad \times\mathbf{E}_{g_I^n}\left\{\left[\frac{a}{3}+\frac{1}{3}c_I-\frac{2c_E}{3}-\frac{F_E(c_E)}{f_E(c_E)}\right]\left[\frac{a}{3}+\frac{1}{3}c_I-\frac{2c_E}{3}\right]\right\} \\
&= \quad \sum_{n=1}^{N}\int_0^1 x_E(c_E)f_E(c_E)dc_E\int_{\mathcal{C}_E^n}h(c_E)dc_E \times \\
&\quad \times\int_{\mathcal{C}_E^n}\left\{\begin{array}{c}\left[\frac{a}{3}+\frac{1}{3}c_I-\frac{2c_E}{3}-\frac{F_E(c_E)}{f_E(c_E)}\right]\left[\frac{a}{3}+\frac{1}{3}c_I-\frac{2c_E}{3}\right]\times \\ \frac{x_E(c_E)f_E(c_E)}{\int_{\mathcal{C}_E}x_E(c_E)f_E(c_E)dc_E\int_{\mathcal{C}_E^n}h(c_E)dc_E}\end{array}\right\}dc_E \\
&= \quad \int_{\mathcal{C}_E}\left\{\left[\frac{a}{3}+\frac{1}{3}c_I-\frac{2c_E}{3}-\frac{F_E(c_E)}{f_E(c_E)}\right]\left[\frac{a}{3}+\frac{1}{3}c_I-\frac{2c_E}{3}\right]\right\}x_E(c_E)f_E(c_E)dc_E \qquad (50)
\end{aligned}
$$

Note that the inequality following for any $x_E(c_E)$, which means regardless of the allocation rule, it is always the best to fully reveal the entrant's production cost to the incumbent.

Define

$$\tilde{J}_E(c_E) = \left[\frac{a}{3} + \frac{1}{3}c_I - \frac{2c_E}{3} - \frac{F_E(c_E)}{f_E(c_E)}\right]\left[\frac{a}{3} + \frac{1}{3}c_I - \frac{2c_E}{3}\right] \tag{51}$$

It measures the marginal revenue of issuing the object to the entrant and fully revealing the entrant's production cost to the incumbent. Note that the term $\frac{a}{3} + \frac{1}{3}c_I - \frac{2c_E}{3}$ is always strictly positive according to Assumption 1. The term $\frac{a}{3} + \frac{1}{3}c_I - \frac{2c_E}{3} - \frac{F_E(c_E)}{f_E(c_E)}$ is strictly decreasing in c_E, and is strictly positive at $c_E = 0$. We let c_E^* be the point that the above term crosses zero if $\frac{a}{3} + \frac{1}{3}c_I - \frac{2}{3} - \frac{1}{f_E(1)} < 0$, and be 1 if $\frac{a}{3} + \frac{1}{3}c_I - \frac{2}{3} - \frac{1}{f_E(1)} \geq 0$. We thus obtain

$$R_N \;\leq\; \int_0^{c_E^*} \tilde{J}_E(c_E)dF_E(c_E)$$

As a result, we have established an upper bound revenue for the designer. If we can construct a feasible mechanism in the original problem that achieves this upper bound revenue, then it should be an optimal mechanism. The following proposition shows that this upper bound revenue is always achievable.

Proposition 2 *The following mechanism maximizes the designer's revenue.*

(i) Allocation rule:

$$x_E(v_E) = \begin{cases} 1, & if\ 0 \leq c_E \leq c_E^*; \\ 0, & if\ c_E^* < c_E \leq 1; \end{cases} \tag{52}$$

(ii) Aftermarket production recommendations:

$$q_I(c_E) = \frac{a - 2c_I + c_E}{3} \tag{53}$$

$$q_E(c_E) = \frac{a + c_I - 2c_E}{3} \tag{54}$$

(iii) The entrant's transfer payment to the designer:

$$t_E(c_E) = \begin{cases} \left(\frac{a+c_I-2c_E}{3}\right)^2 - \int_{c_E}^{c_E^*} \left(\frac{a+c_I-2\xi}{3}\right) d\xi, & if\ 0 \leq c_E \leq c_E^*; \\ 0, & if\ c_E^* < c_E \leq 1; \end{cases}, \tag{55}$$

(iv) The designer's revenue:

$$R_N = \int_0^{c_E^*} \tilde{J}_E(c_E) dF_E(c_E)$$

Proof: It is easy to verify that the above mechanism generates the upper bound revenue. We only need to prove that it satisfies the feasibility constraints. Since the aftermarket recommendation $q_I(c_E)$ is a strictly increasing function, upon seeing the recommendation, the incumbent will infer exactly the entrant's production cost. Therefore, g_I^n is a degenerated distribution at the true value of c_E. Lemma 3 then confirms that IC_I^A and IC_E^A are satisfied. Now consider IC_E^P and PC_E. When $\tilde{c}_E > c_E^*$,

$$U_E(c_E, \tilde{c}_E) = 0 \tag{56}$$

When $\tilde{c}_E < c_E^*$,

$$U_E(c_E, \tilde{c}_E) = \left[\frac{a - \frac{a-2c_I+\tilde{c}_E}{3} - c_E}{2}\right]^2 - \left(\frac{a + c_I - 2\tilde{c}_E}{3}\right)^2 + \int_{\tilde{c}_E}^{c_E^*} \frac{a + c_I - 2\xi}{3} d\xi \tag{57}$$

$$\begin{aligned}
\frac{\partial U_E(c_E, \tilde{c}_E)}{\partial \tilde{c}_E} &= 2(-\frac{1}{6})\left[\frac{a - \frac{a-2c_I+\tilde{c}_E}{3} - c_E}{2}\right] - 2(-\frac{2}{3})\left(\frac{a + c_I - 2\tilde{c}_E}{3}\right) - \frac{a + c_I - 2\tilde{c}_E}{3} \\
&= (-\frac{1}{3})\left[\frac{2a + 2c_I - \tilde{c}_E - 3c_E}{6}\right] + \frac{1}{3}\left(\frac{a + c_I - 2\tilde{c}_E}{3}\right) \\
&= -\left[\frac{2a + 2c_I - \tilde{c}_E - 3c_E}{18}\right] + \left(\frac{2a + 2c_I - 4\tilde{c}_E}{18}\right) \\
&= \left[\frac{-\tilde{c}_E + c_E}{6}\right] \qquad (58)
\end{aligned}$$

$$\frac{\partial^2 U_E(c_E, \tilde{c}_E)}{\partial \tilde{c}_E{}^2} = -\frac{1}{6} \tag{59}$$

Thus, $\tilde{c}_E = c_E$ is a maximum and truthfully reporting is optimal, i.e., IC_E^P is satisfied. PC_E is satisfied since $U_E(1, 1) = 0$. **Q.E.D.**

Similarly, we can summarize some properties of the optimal mechanism in some corollaries. All of the corollaries in the partial control scenario continue to hold. In addition, the outcome in the aftermarket is the same as if the entrant and the incumbent were in Cournot competitions under complete information.

The following corollaries sum up some additional properties. When the designer has no control over the aftermarket, a simple and popular mechanism is to make a take-it-or-leave-it to the entrant. However, such mechanism will not be able to elicit all the information from the entrant. With the take-it-or-leave-it offer, the designer can only infer whether the entrant's production cost is above or below a cutoff instead of its exact value. As a result, we have

Corollary 5 *A take-it-or-leave-it offer can never be revenue maximizing for the designer.*[13]

In both scenarios, entry happens only when the entrant's production cost is lower than a cutoff. If we compare the cutoffs in the two scenario, we obtain $c_E^* \leq \hat{c}_E$, which implies

Corollary 6 *Entry happens more often under partial control than under no control.*

The intuition is that when the designer has more control over the entrant, she can extract more surplus from the entrant, and therefore, she is more willing to let the entrant to produce. Furthermore, if we compare the revenues between the no control scenario and partial control scenario, we obtain

Corollary 7 *The revenue is strictly higher under partial control than under no control.*

This is because when the designer has partial control, at least she can implement the same mechanism that is optimal under no control. Comparing (23) and (56), we conclude that they are not equal. It is easy to show that when the designer can also dictate a production level for the incumbent, she can achieve even higher revenue, which is a standard regulation problem. This suggests that the moral hazard problem does limit the designer's rent extraction ability even with risk neutral agents, in contrast to the previous literature such as McAfee and McMillian [12].

[13] In the partial control scenario, the designer needs to choose the entrant's production level, and it sounds strange to mention take-it-or-leave-it offers. Obviously, a take-it-or-leave-it offer is not optimal in that situation.

5 Extension to I entrants

The restriction to a single entrant is only for expositional simplicity. The analysis can be easily extended to allow I entrants. Here, we focus on the no control scenario, and the partial control scenario is similar. The marginal revenue of allocating the object to entrant i is simply $\tilde{J}_i(c_i)$ by replacing subscript E to i in equation (51). Therefore, the designer simply allocates the object to the entrant with the highest $\tilde{J}_i(c_i)$ if it is positive. The recommendations remain the same and the incumbent will infer the exact production cost of the winning entrant in the aftermarket.

When $I = 1$, we have illustrated that the commonly observe mechanism, i.e., a take-it-or-leave-it offer, is suboptimal. However, when $I \geq 2$, the optimal mechanism can be implemented by a simple and commonly adopted mechanism. Note that $\tilde{J}_i(c_i)$ is strictly decreasing when it is positive. Therefore, if the entrants are symmetric, it is in the designer's interest to allocate the object to the firm with the lowest cost, conditional on it is lower than the cutoff c_i^*. We thus have

Proposition 3 *Under the no control scenario, when there are multiple symmetric entrants, a first-price auction with a reserve price and the announcement of the transaction price implements the optimal mechanism.*

When the entrants adopt symmetric decreasing bidding function in the auction, the entrant with the lowest cost wins. Furthermore, from the transaction price, the incumbent will infer the winning entrant's exact production cost, and the Cournot competition is as if under complete information. In contrast, second-price auctions with the announcement of the transaction price cannot implement the optimal mechanism, as the transaction price only contains information about the second lowest production cost and the winner's production cost remains uncertain.

6 Discussions and Conclusions

In this paper, we study the optimal mechanism design problem with aftermarket competition in which the designer has perfect control of the primary market but not the aftermarket. The designer sells an object (franchise, licence, etc) to entrants for operating in an industry currently occupied by an incumbent. While the incumbent's production cost is commonly known, the entrants' production costs are privately informed. We fully characterize the optimal mechanisms for two scenario: partial control and no control. The optimal mechanism are deterministic and the designer fully reveals the winning entrant's production cost to the incumbent under both scenarios. In addition, in the no control scenario, if there is a single entrant, it is never optimal for the designer to make a take-it-or-leave-it offer to the entrant; meanwhile, when there are multiple symmetric entrants, the optimal mechanism can be implemented by a first-price auction with a reserve price and the announcement of the winning bid. Entry happens more often and the designer can achieve strictly more revenue in the partial control scenario.

The model can be extended in several directions. First, the incumbent may also have private cost. Second, the aftermarket competition may not need to be Cournot competition. Bertrand or Stackelberg competitions may be applicable for different industries. It would be interesting to extend the model to accommodate a general abstract aftermarket competition or even more general aftermarket interactions such as resale and collusion. While the current paper illustrates that, under the assumptions of our paper, it is optimal to fully reveal information to the aftermarket, one may ask whether this is true for any aftermarket interactions.

The answer is negative. In Zhang and Wang [17], the aftermarket interaction is resale, and it is found that fully concealing the information is optimal. It is thus a more subtle question on how to reveal the information to the aftermarket in a general setup. Third, the entire market structure could be modeled differently. For example, both the entrant and the incumbent could interact in the primary market. Fourth, the regulation literature usually assumes full controlling power by the regulator and no moral hazard problem. It would be interesting to reexamine the same issues with imperfect regulator power in the presence of moral hazard problem. Finally, the objective of the designer could be different from just maximizing revenue. We leave these open questions to future investigations.

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